Useful Formulas

Measures of Risk

Variance of returns: $\sigma^2 = \sum_s p(s)[r(s) - E(r)]^2$

Standard deviation: $\sigma = \sqrt{\sigma^2}$

Covariance between returns: $\text{Cov}(r_i, r_j) = \sum_s p(s)[r_i(s) - E(r_i)]\,[r_j(s) - E(r_j)]$

Beta of security i: $\beta_i = \dfrac{\text{Cov}(r_i, r_M)}{\text{Var}(r_M)}$

Portfolio Theory

Expected rate of return on a portfolio with weights w_i in each security: $E(r_p) = \sum_{i=1}^{n} w_i E(r_i)$

Variance of portfolio rate of return: $\sigma_p^2 = \sum_{j=1}^{n} \sum_{i=1}^{n} w_j w_i \,\text{Cov}(r_i, r_j)$

Market Equilibrium

The security market line: $E(r_i) = r_f + \beta_i[E(r_M) - r_f]$

Fixed-Income Analysis

Present value of \$1:

Discrete period compounding: $\text{PV} = 1/(1 + r)^T$

Continuous compounding: $\text{PV} = e^{-rT}$

Forward rate of interest for period T: $f_T = \dfrac{(1 + y_T)^T}{(1 + y_{T-1})^{T-1}} - 1$

Real interest rate: $r = \dfrac{1 + R}{1 + i} - 1$

where R is the nominal interest rate and i is the inflation rate

Duration of a security: $D = \sum_{t=1}^{T} t \times \dfrac{CF_t}{(1 + y)^t}$ /Price

Modified duration: $D^* = D/(1 + y)$

Equity Analysis

Constant growth dividend discount model: $V_0 = \dfrac{D_1}{k - g}$

Sustainable growth rate of dividends: $g = \text{ROE} \times b$

Price/earnings multiple: $P/E = \dfrac{1 - b}{k - \text{ROE} \times b}$

$$\text{ROE} = (1 - \text{Tax rate})\left[\text{ROA} + (\text{ROA} - \text{Interest rate})\frac{\text{Debt}}{\text{Equity}}\right]$$

Derivative Assets

Put-call parity: $P = C - S_0 + PV(X + \text{dividends})$

Black-Scholes formula: $C = Se^{-\delta T}N(d_1) - Xe^{-rT}N(d_2)$

$$d_1 = \frac{\ln(S/X) + (r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Spot-futures parity: $F_0 = S_0(1 + r - d)^T$

Interest rate parity: $F_0 = E_0\left(\dfrac{1 + r_{\text{US}}}{1 + r_{\text{foreign}}}\right)^T$

Performance Evaluation

Sharpe's measure: $S_p = \dfrac{\overline{r}_p - \overline{r}_f}{\sigma_p}$

Treynor's measure: $T_p = \dfrac{\overline{r}_p - \overline{r}_f}{\beta_p}$

Jensen's measure, or alpha: $\alpha_p = \overline{r}_p - [\overline{r}_f + \beta_p(\overline{r}_M - \overline{r}_f)]$

Geometric average return: $r_G = [(1 + r_1)(1 + r_2)\ldots(1 + r_T)]^{1/T} - 1$